to the finite-thickness of the vane trailing edges.

It can be seen from Fig. 3 that the test using the slider with the radiused leading and trailing edges resulted in a generally higher discharge coefficient than was obtainable with the sharp edged slider, but only at the expense of vectoring capability. The vectoring range was greatest using the slider with sharp leading and trailing edges in conjunction with the airfoil-shaped vanes.

The experimentally obtained C_D-slider position relationships, for the uniform aperture width model, allow first approximations to be made of the required vane-span distributions necessary to ensure that a constant effective exit area will prevail over the range of operation in which the entire flow passes through the ventral opening. This has been done for the case of greatest practical interest, that with airfoil cross-section vanes used in conjunction with the sharp-edged slider; the result is shown in Fig. 4.

Conclusions

It was concluded that in general, vectoring nozzle with a slide-valve-controlled ventral opening appears to be feasible from the viewpoint of internal and external aerodynamics. It was further concluded that a range of vectoring between 10° forward to 30° aft of vertical with the entire flow passing through the ventral exit, essential for efficient operation of the nozzle and adequate aircraft control, is well within the capabilities of the device. A prediction of the thrust coefficient, based on realistic operating conditions (critical pressure ratio, Mach No. = 0.3 in the jet pipe), of the most practical configuration featuring airfoil cross-section vanes and a sharp-edged slider yielded a value of 0.95. Allowances were made for losses due to flow turning in the stream approaching the nozzle guidevanes, an effect absent from the model, flow over the guide vanes and divergence of the jet; the latter was based on observations of the flow paths in the efflux of the model nozzle.

Focussing of the thrust vectors, as depicted elsewhere,[2] was not investigated in the experiments. Failure to achieve a precise, singular, focussing effect may, or may not, be desirable: it would result in the focus of the thrust vectors tracing out a locus, which would be a function of slider position, instead of possessing a unique location independent of slider position. It would require more sophisticated tests than those reported here to investigate rigorously this aspect of the performance.

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Vertical Tail Size Needed for a Coordinated Turn Reversal

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THIS Note presents a simplified analysis of the vertical tail load required to reverse the direction of a coordinated turn. The load is proportional to the product of the tip helix angle and the aircraft weight, but is independent of the speed of flight. It may therefore set a lower limit on the acceptable vertical tail size for satisfactory turn coordination in low-speed flight, particularly for sailplanes and STOL airplanes.

Presented as Paper 74-1004 at the AIAA/MIT/SSA 2nd International Symposium on the Science and Technology of Low Speed and Motorless Flight, Cambridge, Massachusetts, September 11-13, 1974; submitted February 14, 1975; revision received March 26, 1975.

Index category: Aircraft Handling, Stability, and Control.

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Analysis

The rate of change of heading in a coordinated level turn is given by

$$\mathrm{d}\psi/\mathrm{d}t = g \tan\phi \; V \tag{1}$$

where ψ and ϕ are the usual Euler angles, V is the flight speed, and g is the acceleration of gravity. During turn reversals the time derivative of the heading rate is therefore

$$\mathrm{d}^2\psi/\mathrm{d}t^2 = (g/V)\sec^2\phi(\mathrm{d}\phi/\mathrm{d}t)$$

As the aircraft rolls through wings level attitude, the rate of change of heading becomes identical to the angular acceleration about the Z stability axis, and the vertical tail must supply a moment

$$Y_v\ell_v = -I_z(\mathrm{d}R/\mathrm{d}t) = -mk_Z{}^2(g/V)\;(\mathrm{d}\phi/\mathrm{d}t) \tag{2}$$

where Y_v is the tail load (positive to starboard), ℓ_v is the tail moment arm, k_Z is the radius of gyration about the aircraft Z axis, and R is the yaw rate.

The rolling wing will generate a yawing moment given by

$$N = \tfrac{1}{2}\rho V^2 Sb\left[C_{n_\delta}\delta + C_{n_p}\frac{b}{2V}\frac{\mathrm{d}\phi}{\mathrm{d}t}\right]$$

To the extent that the lift distribution remains nearly elliptic during rolling (the additional lift due to rolling approximately cancels the additional lift due to control deflection), and to the extent that the lift per unit span, $\mathrm{d}L/\mathrm{d}y$, at a typical spanwise station, y, is rotated through the local helix angle (as would be true of wing sections having good leading-edge flow), the yawing moment may be approximated by

$$N = \tfrac{1}{2}\rho V^2 Sb\left[-\frac{C_L}{8}\right]\frac{b}{2V}\frac{\mathrm{d}\phi}{\mathrm{d}t} \tag{3}$$

and this additional (adverse) yawing moment also has to be supplied by the vertical tail. The total tail load turn reversals is therefore

$$Y_v = -\frac{1}{\ell_v}\left[mk_Z{}^2\frac{g}{V}\frac{\mathrm{d}\phi}{\mathrm{d}t}\right.$$
$$\left. + \tfrac{1}{2}\rho V^2 SB(1/8C_L)(b/2V)(\mathrm{d}\phi/\mathrm{d}t)\right]$$

or

$$Y_v = -mg(b/\ell_v)[2(k_Z/b)^2 + \tfrac{1}{8}]\;[(b/2V)\;(\mathrm{d}\phi/\mathrm{d}t)] \tag{4}$$

Alternatively, Eq. (4) may be expressed as a vertical tail volume coefficient requirement

$$(VTVC) = \frac{C_L}{C_{L_V}}\frac{q}{q_v}[2(k_z/b)^2 + 0.125][(b/2V)(\mathrm{d}\phi/\mathrm{d}t)] \tag{5}$$

where C_L is the airplane lift coefficient, C_{L_V} is the maximum lift coefficient of the vertical tail, q is the flight dynamic pressure, and q_v is the dynamic pressure averaged over the vertical tail.

Discussion

A typical light airplane has the following characteristics:

$$\ell_v/b = 0.45,\; k_Z/b = 0.25$$

$$\left[\frac{b}{2V}\frac{d\phi}{dt}\right]_{max} = 0.1 \text{ (full aileron deflection)}$$

According to Eq. (4) the tail load in a full aileron deflection level turn reversal is

$$\frac{Y_v}{mg} = \frac{1}{0.45}\,[2(0.25)^2 + 0.125]\,(0.1) = 0.055,55...$$

and the vertical tail size needed for turn reversal at $C_{L_{max}}$ should be 5-6% of the wing area if the maximum lift coefficient of the vertical tail with deflected rudder is the same as that of the wing. Unflapped single engine light airplanes of the 1930's were in fact provided with vertical tails of this size.

If such an airplane is modified to incorporate an 'STOL conversion'' which doubles or triples the maximum lift coefficient of the wing compared to the vertical tail, Eq. (4) would indicate that the vertical tail size should be doubled or tripled accordingly. Failure to increase the vertical tail size for such an STOL conversion can lead to large excursions in side-slip during ''side step'' maneuvers on final approach at high C_L with the very real possibility of inadvertent asymmetric stall, loss of control, and crash during the post-stall gyration.

Equation (4) would also indicate that pilots should have difficulty performing low-speed turn reversals with short-tailed aircraft such as sailplanes or ''tailless'' airplanes, even with heavy rudder ''coordination,'' and I believe they do. The relation apparently provides rationalization for Koppen's rule of thumb: ''No airplane with less than a semispan vertical tail arm ever had good stalling properties.''

Feedback/Feedforward Matrices for Optimal Following of a Forced Model

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Introduction

DURING the last decade, many papers have considered the design of model reference systems with optimal control theory.[1-8] Most developments apply to the first group of them, that use a *real model* (both free and forced) as a kind of prefilter to the plant. The second group with *implicit model* is not as well documented, and relations for the controller gains are given in general for the free model only. This letter fills the gap by deriving differential equations for the controller matrices when the model also has an input vector.

Problem Statement

Consider the linear constant-coefficients system

$$\mathring{x}_p = A_p x_p + B_p u_p \tag{1}$$

where x_p is the n-vector of states, u_p is the m-vector of inputs, A_p and B_p are matrices of corresponding dimensions. A control u_p is sought such that the derivatives x_p behave in a determined way

$$\mathring{x}_d = A_m x_p + B_m u_m \tag{2}$$

chosen after the equation of a model

$$\mathring{x}_m = A_m x_m + B_m u_m \tag{3}$$

$$\mathring{u}_m = D u_m \tag{4}$$

The objective is to minimize the quadratic performance index

$$I = \tfrac{1}{2}\int_0^T [(\mathring{x}_p - \mathring{x}_d)^T Q(\mathring{x}_p - x_d) + u_p^T R u_p]\, dt \tag{5}$$

The usual methods of optimal control theory can be used to compute the control u of a linear system

$$\mathring{x} = Ax + Bu \tag{6}$$

that minimizes

$$J = \tfrac{1}{2}\int_0^T (\mathring{x}^T Q_o \mathring{x} + u^T R u)\, dt \tag{7}$$

The Hamiltonian is formed with a vector λ of the costates

$$H = \tfrac{1}{2}x^T A^T Q_o A x + x^T A^T Q_o B u + \tfrac{1}{2}u^T[R + B^T Q_o B]\,u + \lambda^T[Ax + Bu] \tag{8}$$

Since there are no constraints on u, the extremal control is given by $\partial H/\partial u = 0$ and reads

$$u^* = -\tilde{R}^{-1}B^T[\lambda + Q_o A x] \tag{9}$$

where

$$\tilde{R} = R + B^T Q_o B \tag{10}$$

The extremal Hamiltonian may then be written

$$H^* = \tfrac{1}{2}x^T A^T \tilde{Q}_o A x - \lambda^T \tilde{A} x - \tfrac{1}{2}{}^T b \tilde{R}^{-1} B^T \lambda \tag{11}$$

with

$$\tilde{Q}_o = Q_o - Q_o B \tilde{R}^{-1} B^T Q_o \tag{12}$$

$$\tilde{A} = A - B\tilde{R}^{-1}B^T Q_o A \tag{13}$$

In the canonical equations

$$\partial H^*/\partial x = -\mathring{\lambda} = A^T \tilde{Q}_o A x + A^T \lambda \tag{14a}$$

$$\partial H^*/\partial \lambda = \mathring{x} = \tilde{A}x - B\tilde{R}^{-1}B^T\lambda \tag{14b}$$

the costate λ can be eliminated by assuming $\lambda = Px$, where P is a time-varying matrix, given by a Riccati equation

$$\mathring{P} + P\tilde{A} + \tilde{A}^T P - PB\tilde{R}^{-1}B^T P + A^T \tilde{Q}_o A = 0 \tag{15}$$

$$P(T) = 0$$

The control law (9) is then

$$u^* = -\tilde{R}^{-1}B^T[P + Q_o A]\,x \tag{16}$$

Application to the Specific Problem

The problem stated in Ref. 2 can be brought to the form of Ref. 3 by the choice of

$$x = (x_p^T, x_d^T, u_m^T)^T \tag{17a}$$

$$u = u_p \tag{17b}$$

Received March 28, 1975.
Index categories: Aircraft Handling, Stability, and Control; Navigation, Control, and Guidance Theory.
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